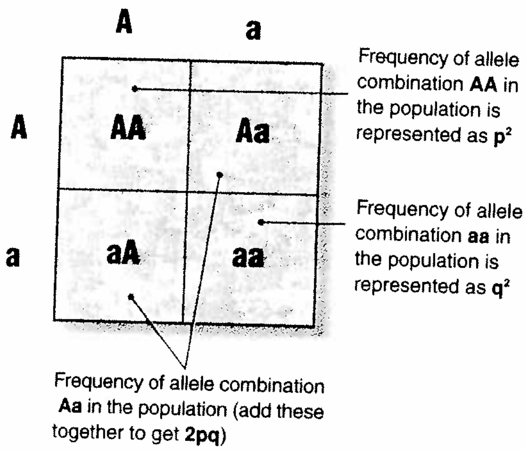
Calculating Allele Frequencies in Populations

The **Hardy-Weinberg equation** provides a simple mathematical model of genetic equilibrium in a gene pool, but its main application in population genetics is in calculating allele and genotype frequencies in populations, particularly as a means of studying changes and measuring their rate. The use of the Hardy-Weinberg equation is described below.

Punnett square



$$(p + q)^2 = p^2 + 2pq + q^2 = 1$$

Frequency of allele types	**Frequency of allele combinations**
p = Frequency of allele A	p^2 = Frequency of AA (homozygous dominant)
q = Frequency of allele a	2pq = Frequency of Aa (heterozygous)
	q^2 = Frequency of aa (homozygous recessive)

The Hardy-Weinberg equation is applied to populations with a simple genetic situation: dominant and recessive alleles controlling a single trait. The frequency of all of the dominant (A) and recessive alleles (a) equals the total genetic complement, and adds up to 1 or 100% of the alleles present.

How To Solve Hardy-Weinberg Problems

In most populations, the frequency of two alleles of interest is calculated from the proportion of homozygous recessives (q^2), as this is the only genotype identifiable directly from its phenotype. If only the dominant phenotype is known, q^2 may be calculated (1 – the frequency of the dominant phenotype). The following steps outline the procedure for solving a Hardy-Weinberg problem:

***Remember that all calculations must be carried out using proportions,* NOT PERCENTAGES!**

1. Examine the question to determine what piece of information you have been given about the population. In most cases, this is the percentage or frequency of the homozygous recessive phenotype q^2, or the dominant phenotype $p^2 + 2pq$ (see note above).
2. The first objective is to find out the value of p or q, If this is achieved, then every other value in the equation can be determined by simple calculation.
3. Take the square root of q^2 to find q.
4. Determine p by subtracting q from 1 (i.e. p = 1 – q).
5. Determine p^2 by multiplying p by itself (i.e. $p^2 = p \times p$).
6. Determine 2pq by multiplying p times q times 2.
7. Check that your calculations are correct by adding up the values for $p^2 + q^2 + 2pq$ (the sum should equal 1 or 100%).

Worked example

Among Caucasians in the USA, approximately 70% of people can taste the chemical phenylthiocarbamide (PTC) (the dominant phenotype), while 30% are non-tasters (the recessive phenotype).

Determine the frequency of:	***Answers***
(a) Homozygous recessive phenotype(q^2).	30% - provided
(b) The dominant allele (p).	45.2%
(c) Homozygous tasters (p^2).	20.5%
(d) Heterozygous tasters (2pq).	49.5%

Data: The frequency of the dominant phenotype (70% tasters) and recessive phenotype (30% non-tasters) are provided.

Working:

Recessive phenotype: q^2 = 30%
use 0.30 for calculation

therefore: q = 0.5477
square root of 0.30

therefore: p = 0.4523
1 – q = p
1 – 0.5477 = 0.4523

Use p and q in the equation (top) to solve any unknown:

Homozygous dominant p^2 = 0.2046
(p x p = 0.4523 x 0.4523)

Heterozygous: 2pq = 0.4953

1. A population of hamsters has a gene consisting of 90% M alleles (black) and 10% m alleles (gray). Mating is random.
Data: Frequency of recessive allele (10% m) and dominant allele (90% M).

Determine the proportion of offspring that will be black and the proportion that will be gray (show your working):

Recessive allele: q =

Dominant allele: p =

Recessive phenotype: q^2 =

Homozygous dominant: p^2 =

Heterozygous: 2pq =

2. You are working with pea plants and found 36 plants out of 400 were dwarf.
Data: Frequency of recessive phenotype (36 out of 400 = 9%)

(a) Calculate the frequency of the tall gene: ______________________

(b) Determine the number of heterozygous pea plants:

____

____

Recessive allele:	q =	
Dominant allele:	p =	
Recessive phenotype:	q^2 =	
Homozygous dominant:	p^2 =	
Heterozygous:	$2pq$ =	

3. In humans, the ability to taste the chemical phenylthiocarbamide (PTC) is inherited as a simple dominant characteristic. Suppose you found out that 360 out of 1000 college students could not taste the chemical.
Data: Frequency of recessive phenotype (360 out of 1000).

(a) State the frequency of the gene for tasting PTC:

____

(b) Determine the number of heterozygous students in this population:

____

Recessive allele:	q =	
Dominant allele:	p =	
Recessive phenotype:	q^2 =	
Homozygous dominant:	p^2 =	
Heterozygous:	$2pq$ =	

4. A type of deformity appears in 4% of a large herd of cattle. Assume the deformity was caused by a recessive gene.
Data: Frequency of recessive phenotype (4% deformity).

(a) Calculate the percentage of the herd that are carriers of the gene:

____

(b) Determine the frequency of the dominant gene in this case:

____

Recessive allele:	q =	
Dominant allele:	p =	
Recessive phenotype:	q^2 =	
Homozygous dominant:	p^2 =	
Heterozygous:	$2pq$ =	

5. Assume you placed 50 pure bred black guinea pigs (dominant allele) with 50 albino guinea pigs (recessive allele) and allowed the population to attain genetic equilibrium (several generations have passed).
Data: Frequency of recessive allele (50%) and dominant allele (50%).

Determine the proportion (%) of the population that becomes white:

____

Recessive allele:	q =	
Dominant allele:	p =	
Recessive phenotype:	q^2 =	
Homozygous dominant:	p^2 =	
Heterozygous:	$2pq$ =	

6. It is known that 64% of a large population exhibit the recessive trait of a characteristic controlled by two alleles (one is dominant over the other).
Data: Frequency of recessive phenotype (64%). Determine the following:

(a) The frequency of the recessive allele: ______________________

(b) The percentage that are heterozygous for this trait: ______________________

(c) The percentage that exhibit the dominant trait: ______________________

(d) The percentage that are homozygous for the dominant trait: ______________________

(e) The percentage that has one or more recessive alleles: ______________________

7. Albinism is recessive to normal pigmentation in humans. The frequency of the albino allele was 10% in a population.
Data: Frequency of recessive allele (10% albino allele).

Determine the proportion of people that you would expect to be albino:

____

____

____

Recessive allele:	q =	
Dominant allele:	p =	
Recessive phenotype:	q^2 =	
Homozygous dominant:	p^2 =	
Heterozygous:	$2pq$ =	

more significant for small populations than it is for large populations.

2. Microevolution is changes to the allele frequencies within a population. Natural selection, genetic drift, gene flow, and mutation are all processes that can directly affect gene frequencies in a population, and result in microevolution. Microevolution can occur over a short period of time (e.g. between one generation and the next), resulting in rapid change of allele frequencies within a population. The development of drug resistance in bacteria is often used as an example of microevolution. The following factors (or a combination of them) explain how drug resistance could occur:
   - Natural selection favored a gene for resistance against a particular drug.
   - The population received new immigrants carrying the gene for drug resistance.
   - Mutation produced a gene that was drug resistant.
   - Random genetic drift altered the allele frequencies of the population so that the presence of the drug resistant gene was higher.

3. (a) Increase genetic variation: Gene flow (migration), large population size, mutation.
   (b) Decrease genetic variation: Natural selection, non-random mating (mate choice), genetic drift.

Sexual Selection (page 243)

1. **Intrasexual selection** involves competition within one sex (usually males) with the winner gaining access to the opposite sex. Intersexual selection (or **mate choice**) also involves competition (usually involving displays) between members of one sex (usually males) for the opposite sex (usually the females) but it is the opposite sex who chooses their mate.

2. Sexual selection results in marked **sexual dimorphism** because the competitive gender (usually males) have to advertise their superiority as a mate to rivals and potential suitors. They do this by means of elaborate ornamentation (e.g. horns, antlers, plumage) and behavior (ritualized fighting, stereotyped courtship displays). The development of these characteristics leads to increasing divergence in appearance between the two sexes.

Changes in a Gene Pool (page 244)

1. The dominant (A)allele produces pigment. Beetles with two recessive (aa) alleles are pale in color because no pigment is produced. The presence of either a single dominant allele (Aa) or two dominant alleles (AA) produces pigment, so the beetles are black in color.

2. This exercise (a)-(c) demonstrates how the allele frequencies change as different of event take place:

Phase 1: Initial gene pool
This is the gene pool before any of the events take place:

	A	a	AA	Aa	aa
No.	27	23	7	13	5
%	54	46	28	52	20

Phase 2: Natural selection
The population is now reduced by 2 to 23. The removal of two homozygous recessive individuals has altered the allele combination frequencies (rounding errors occur).

	A	a	AA	Aa	aa
No.	27	19	7	13	3
%	58.7	41.3	30.4	56.5	13.0

Phase 3: Immigration / emigration
The addition of dominant alleles and the loss of recessive alleles makes further changes to the allele frequencies.

	A	a	AA	Aa	aa
No.	29	17	8	13	2
%	63	37	34.8	56.5	8.7

Calculating Allele Frequencies in Populations (page 245)

The answers to the panels provided in the workbook are shown below as working. Alternatively, these calculations are quickly done using a spreadsheet.

1. **Working**: q= 0.1, p= 0.9, q^2= 0.01, p^2= 0.81, 2pq= 0.18
   Proportion of black offspring = 2pq + p^2 x 100% = 99%;
   Proportion of gray offspring = q^2 x 100% = 1%

2. **Working**: q= 0.3, p= 0.7, q^2= 0.09, p^2= 0.49, 2pq= 0.42
   (a) Frequency of tall (dominant) gene (allele): 70%
   (b) 42% heterozygous; 42% of 400 = **168**

3. **Working**: q= 0.6, p= 0.4, q^2= 0.36, p^2= 0.16, 2pq= 0.48
   (a) 40% dominant allele
   (b) 48% heterozygous; 48% of 1000 = **480**.

4. **Working**: q= 0.2, p= 0.8, q^2= 0.04, p^2= 0.64, 2pq= 0.32
   (a) 32% heterozygous (carriers)
   (b) 80% dominant allele

5. **Working**: q= 0.5, p= 0.5, q^2= 0.25, p^2= 0.25, 2pq= 0.5
   Proportion of population that becomes white = 25%

6. **Working**: q= 0.8, p= 0.2, q^2= 0.64, p^2= 0.04, 2pq =0.32
   (a) 80% (c) 36% (e) 96%
   (b) 32% (d) 4%

7. **Working**: q= 0.1, p= 0.9, q^2= 0.01, p^2= 0.81, 2pq= 0.18
   Proportion of people expected to be albino (i.e. proportion that are homozygous recessive) = 1%