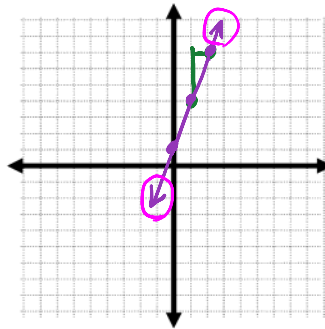# Notes 1

7.1: Slope-Intercept form: $y = mx+b$

Definition of **COEFFICIENT** : the # in front of x ex. __3__x

1. Draw the lines for the following equations:
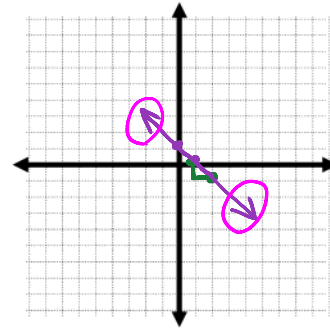
i) $y = 3x + 1$

x	y
0	1
1	4
2	7

ii) $y = -x + 1$ $=-1x+1$

x	y
0	1
1	0
2	-1

a) Calculate the slope of each line.

$$m = \frac{rise}{run} = \frac{3}{1} = 3$$

$$\frac{rise}{run} = \frac{-1}{1} = -1$$

b) What do you notice about the slope and the coefficient on the x variable?

__Coefficient = slope ! They are the same!__

c) What is the y-intercept of each graph above? Graph on left __1__   Graph on right __1__

↳ cross y-axis.

d) What do you notice about the y-intercept and the equation for each graph?

__It is the number at the end of the equation.__

(constant) eg. $y = 3x + 1$

1

SLOPE-INTERCEPT form

When equations are written in slope-intercept form

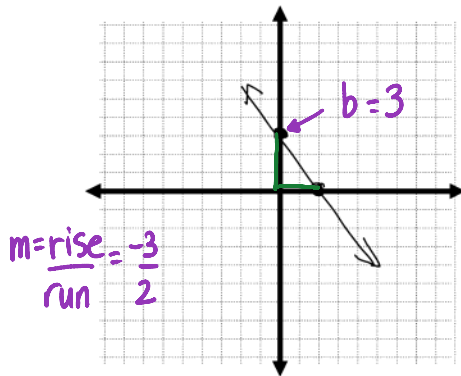$$y = mx + b$$

m is the ___slope___   b is the ___y-intercept___

i) State the slope and y-intecept

		Slope	y-intecept
a)	$y = 4x + 1$	4	1
b)	$y = -1/2x - 10$	$-\frac{1}{2}$	-10
c)	$y = 3x + 0$	3	0

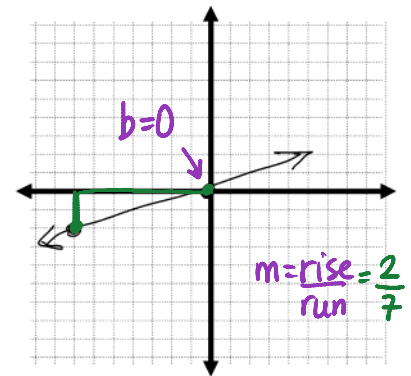ii) Write an equation in slope intercept form given the slope and they y-intercept.

a) Given slope: 2/3 and y-intecept (0, 4) $y = \frac{2}{3}x + 4$

b) Given slope: -4/1 and y-intecept (0, 1) $y = -4x + 1$

iii) Write the equation of the line in y=mx + b form:
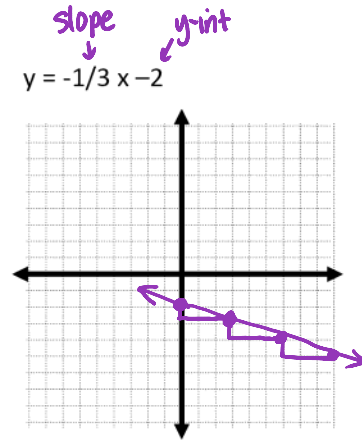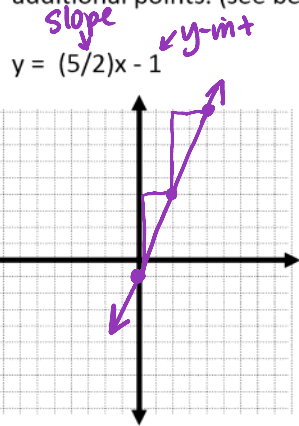
Need
1) slope = m
2) y-int = b

$y = mx + b$

$b = 3$

$m = \dfrac{rise}{run} = \dfrac{-3}{2}$

$b = 0$

$m = \dfrac{rise}{run} = \dfrac{2}{7}$

Equation: $y = \dfrac{-3}{2}x + 3$

Equation: $y = \dfrac{2}{7}x$

2

iv) Graph the following equations without making a table of values.
   Start with the y-intercept and then use the slope to make a 'staircase' from this point to get
   additional points. (see below)

slope   y-int
$y = (5/2)x - 1$

slope   y-int
$y = -1/3 \, x - 2$

v) Equations will often need to be changed into slope-intecept form by solving for y.
   - Move y to one side and everything else to the other side.
   - The equation must end up as "y = $mx+b$ "

a) $3x + y = 4$

   $-3x \qquad -3x$

   $y = -3x + 4$

   slope   y-int

b) $x + 3y = 1$

   $-x \qquad -x$

   $\dfrac{3y}{3} = \dfrac{-x}{3} + \dfrac{1}{3}$

   $y = -\dfrac{1}{3}x + \dfrac{1}{3}$

   slope   y-int

vi) A line has a y-intercepts of -3 and it goes through the points (-1, -7) and (3, 9).
Write the equation of the line in the form y = mx+b.   ①   ②

b

Steps:
1) Calculate the slope using the slope formula.
2) Write the equation in the form y = mx+b, where m is the slope and b is the y-intercept (-3 in
   this case)

$\text{Slope} = \dfrac{\text{rise}}{\text{run}} = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{9 + (+7)}{3 + (+1)}$

$= \dfrac{16}{4}$

$= 4 \leftarrow m$

$y = mx + b$

$\boxed{y = 4x - 3}$

3

7.2: General form: Ax+By+C = 0

General form is another way of writing a line equation: Ax + By + C = 0.   A, B, C = numbers

- A or B cannot both equal zero.
- "A" is a whole number (0, 1, 2, 3, 4, etc...) ** No _Fractions_ !
      A must be positive   or Decimals

Part 1 – Converting an equation to general form using algebra:

$$y = -\frac{2}{3}x + 5 \Big) 3$$ No Fractions

$$3y = -2x + 15$$ A positive $-\frac{2}{3}x \times \frac{3}{1} = -\frac{6x}{3} = {}^-2x$

$$2x + 3y = 15$$

$$\boxed{2x + 3y - 15 = 0}$$

Your Turn: Convert into general form $y = \frac{1}{2}x - 6 \Big) 2$

$$2y = 1x - 12$$
$$-2y \qquad -2y$$
$$\boxed{0 = x - 2y - 12}$$

Part 2 – Converting from the general form to the slope-intercept (y=mx+b) form:

$$3x - 4y + 8 = 0$$ 1) Isolate y term
$$+4y \qquad +4y$$

$$\frac{3x}{4} + \frac{8}{4} = \frac{4y}{4}$$ 2) Divide

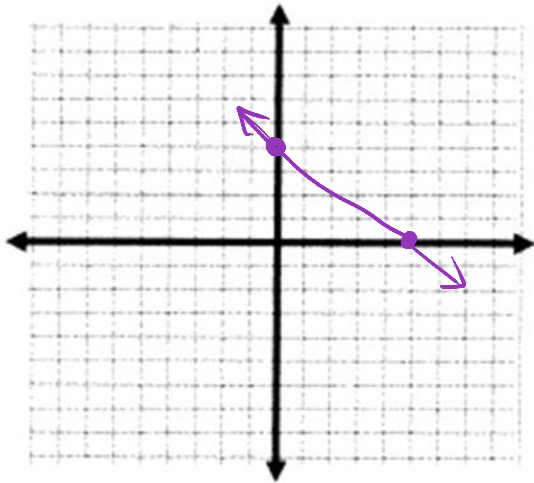$$\frac{3}{4}x + 2 = y \implies \boxed{y = \frac{3}{4}x + 2}$$

Your Turn: Convert into slope-intercept form $2x + 3y - 4 = 0$

$$2x + 3y - 4 = 0$$
$$\qquad -3y \qquad -3y$$

$$\frac{2x}{-3} \frac{-4}{-3} = \frac{-3y}{-3}$$

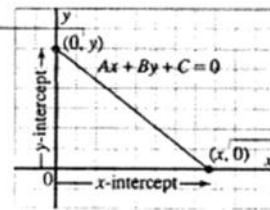$$-\frac{2}{3}x + \frac{4}{3} = y \implies \boxed{y = -\frac{2}{3}x + \frac{4}{3}}$$

4

Part 3 – For the equation $4x + 5y - 20 = 0$

a) Find the x and y intercepts.

b) Use the intercepts to graph the line.

To find the y-intercept, substitute 0 for x then solve for y.

$Ax + By + C = 0$

To find the x-intercept, substitute 0 for y then solve for x.

a) y-int → plug in x=0

$4(0)+5y-20=0$

$5y-20=0$

$\frac{5y}{5} = \frac{20}{5}$

$\boxed{y=4}$

x-int → plug in y=0

$4x+5(0)-20=0$

$4x-20=0$

$\frac{4x}{4} = \frac{20}{4}$

$\boxed{x=5}$

Ex. 1: Find an equation for the straight line passing through B(-1, 5) and C(-3, -1) in the form y=mx+b. Then convert into general form.

①   ②

Solution:

1) Find the slope m using the slope formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-5}{-3+(4)} = \frac{-6}{-2} = 3$$

m = __3__

2) Find the y-intercept b, use y=mx+b and substitute either point B or C for (x,y).

$y = 3x + b$

→ plug in B (-1,5)

$5 = 3(-1)+b$

$5 = -3+b$

$+3 \quad +3$

→ b = 8

b = __8__

Line: __y=3x+8__

$-y \quad -y$

3) Convert to general form.

Line: __0 = 3x-y + 8__

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7.3: Slope-Point form

- We already know that slope = m = $\dfrac{\text{rise}}{\text{run}} = \dfrac{y_2 - y_1}{x_2 - x_1}$

- To get the slope-point form we multiply both sides by $(x - x_1)$

$$y - y_1 = m(x - x_1)$$

Need: 1) slope m
2) any point
(x_1, y_1)

Ex. 1: Use slope-point form to write an equation of a line through (1, -2) and with a slope of 3/4.

$\hookrightarrow y - y_1 = m(x - x_1)$ Need 1) $m = 3/4$
2) point = (1, -2)

$y - (-2) = \dfrac{3}{4}(x - 1)$

$\boxed{y + 2 = \dfrac{3}{4}(x - 1)}$

Ex. 2: Graph the line given an equation in slope-point form.

$y + 1 = -\dfrac{1}{2}(x - 2)$

> To find the slope-point form rewrite the equation so the operations are subtraction.

$y - y_1 = m(x - x_1)$

$y - (-1) = -\dfrac{1}{2}(x - (2))$
$\;\;\;\;\underset{y_1}{}\;\;\;\;\;\;\;\underset{x_1}{}$

1) $m = -\dfrac{1}{2}$

2) point = $(2, -1)$

Ex. 3: Express $y - 3 = 2(x - 1)$ in general form $\rightarrow Ax + By + C = 0$
$\hookrightarrow m = 2$, point: (1, 3)

$y - 3 = 2x - 2$
$-y + 3 \quad\quad -y + 3$

$\boxed{0 = 2x - y + 1}$

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Ex. 4: Find the equation of the line in slope-point form that passes through the points ①(-5,2) and ②(-2, 1)

$$y - y_1 = m(x - x_1)$$

$$y - 1 = -\frac{1}{3}(x - (-2))$$

$$\boxed{y - 1 = -\frac{1}{3}(x + 2)}$$

Need 1) slope $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{1 - 2}{-2 - (-5)} = \dfrac{-1}{3}$

2) point : $(-2, 1)$ (choose 1!)

Ex. 5:   Determine the equation of the line shown on this graph in slope-point and slope-intercept forms.

$$y - y_1 = m(x - x_1)$$

1) $m = \dfrac{rise}{run} = \dfrac{3}{5}$

2) point : $(6, 3)$

$$\boxed{y - 3 = \frac{3}{5}(x - 6)}$$

$$y = mx + b$$

1) $m = \dfrac{3}{5}$

2) y-int = ?

$$y = \frac{3}{5}x + b$$ *plug in (6,3)
→ find b

$$3 = \frac{3}{5}(6) + b$$

$$3 = \frac{18}{5} + b$$
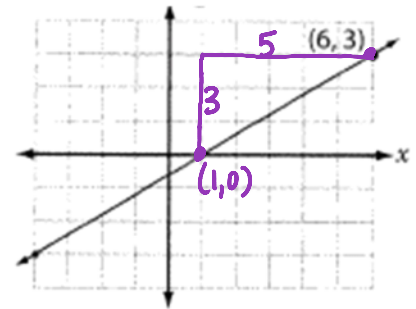
$$3 - \frac{18}{5} = b$$

$$\frac{15}{5} - \frac{18}{5} = b$$

$$-\frac{3}{5} = b$$

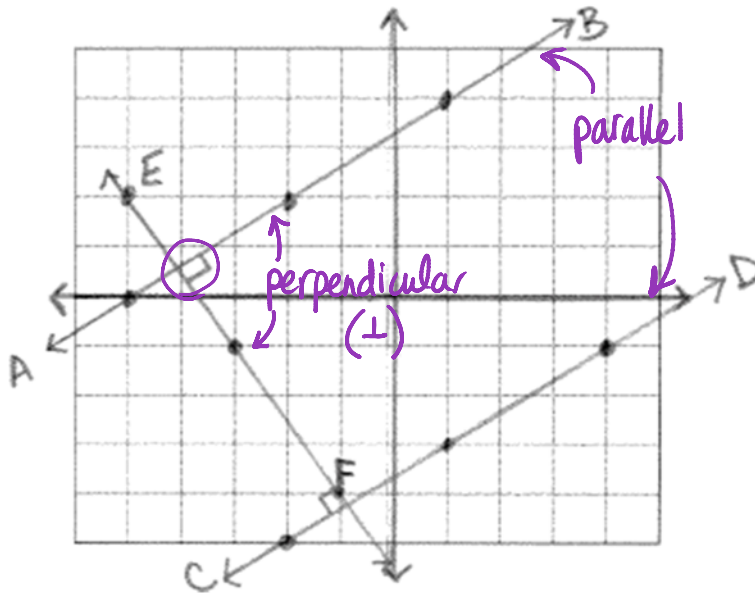$$\boxed{y = \frac{3}{5}x - \frac{3}{5}}$$

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7.4: Parallel and Perpendicular Lines

parallel

perpendicular (⊥)

Calculate the slopes of AB, CD and EF

Slope $AB = \frac{2}{3}$

Slope $CD = \frac{2}{3}$

Slope $EF = -\frac{3}{2}$

- **Parallel lines** have the ____Same____ slope and different y intercepts.

- **Perpendicular lines** have slopes that are __negative__ __reciprocals__ of each other.

   -> Perpendicular lines are at right angles to each other.

Ex. 1: Write the equation of 3 different lines that are <u>parallel</u> to the line $y = \frac{3}{4}x + 4$

$m = 3/4$ ↳ Same slope

$y = 3/4 x + 2$

$y = 3/4 x - 3$ slope   y-int

$y = 3/4 x + 4.425$ $(y = mx + b)$

Ex. 2: The slopes of two <u>parallel</u> line segments are given. Determine the value of x.

↳ same slope!

a) $\frac{x}{4}, 2$

$\cancel{4} \times \frac{x}{\cancel{4}} = 2 \times 4$

$\boxed{x = 8}$

b) $\frac{-x}{3}, \frac{-2}{7}$

$\cancel{3} \times \frac{-x}{\cancel{3}} = \frac{-2}{7} \times \frac{3}{1}$

$-x = -\frac{6}{7}$

$\boxed{x = \frac{6}{7}}$

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→ Same slope

Ex. 3: Write the equation of a line parallel to $y = -3x + 7$ and passes through the (-2, 5)

1) $m = -3$

2) point $= (-2, 5)$

$y = mx + b$

$y - y_1 = m(x - x_1)$

$y - 5 = -3(x - (-2))$

$\boxed{y - 5 = -3(x + 2)}$

Ex. 4: Write a perpendicular slope for each given slope.

a) m = 3/2   perpendicular slope = $\underline{-2/3}$

b) m = -5/2   perpendicular slope = $\underline{2/5}$

c) m = $3/_1$ perpendicular slope = $\underline{-1/3}$

Ex. 5: Identify whether the pair of lines is parallel, perpendicular or neither.

① $y = \frac{3}{4}x + 5$
② $4x + 3y = 6$ } Convert to $y = mx + b$

① $y = \frac{3}{4}x + 5$

$m = \frac{3}{4}$

② $4x + 3y = 6$
     $-4x$ $-4x$
     $\frac{3y}{3} = \frac{-4x}{3} + \frac{6}{3}$

$y = -\frac{4}{3}x + 2$

$m = -\frac{4}{3}$

→ Negative Reciprocal Slopes

$\Rightarrow$ Perpendicular Lines

Ex. 6: Write the equation of a line that passes through the point (-12, -7) and is perpendicular to the line $y = -4x + 7$

↳ neg. rec. slopes

slope $= -\frac{4}{1}$ our slope ($\perp$): $m = \frac{1}{4}$

point: $(-12, -7)$

$y - y_1 = m(x - x_1)$

$y - (-7) = \frac{1}{4}(x - (-12))$

$\boxed{y + 7 = \frac{1}{4}(x + 12)}$
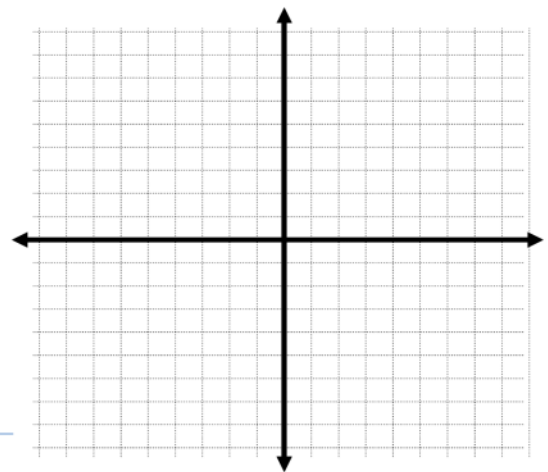
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Practice and Converting Between Line Forms

Example: Consider a line passing through the points (-4, 5) and (6, 0).

   a) Write the equation of this line in slope-point form.

   b) Rewrite the equation in part a) in slope-intercept form.

   c) Rewrite the equation in part a) in general form.

   d) Sketch the graph.

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Concepts	Rate yourself 1 (Low) to 5 (High)	
	Date	
Identify the graph if given the slope and y-intercept.		
Identify the slope and y-intercept if given a graph.		
Determine another point on a line, given the slope and a point on the line.		
Express a linear relation in slope-intercept form (y=mx+b) • If given the slope and y-intercept.		
• If given the slope and one point on the line.		
• If given two points on the line.		
• If given one point on the line and the equation of a parallel or perpendicular line.		
Express a linear relation in general form (Ax + By + C = 0)		
Express a linear relation in point-slope form (y-y1=m(x-x1))		
Convert linear relations between the three forms.		
Graph an equation given in any of the three forms.		
Match a set of linear relations to their graphs.		
Determine whether two lines are parallel or perpendicular.		
Determine whether two equations are equivalent (eg: One given in slope-intercept form and one given in point-slope form)		
Solve problems involving slope, y-intercepts, and equations of lines. (problem-solving)		

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